and the linear function

$$\alpha(\tau) = 1 + \lambda\tau \quad (40)$$

where $\lambda = (b/a_0) - 1$. The viscoelastic solution is compared to the elastic solution for an ablating cylinder with shear modulus $G(0)$.

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A Self-Calibrating Probe for Measuring Atom Concentration in a Hypersonic Flow

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Nomenclature

U = flow velocity
ρ = flow density
μ = viscosity
T = temperature
j = 0 for two-dimensional flow, 1 for three-dimensional flow
Sc = Schmidt number
Pr = Prandtl number
Le = Lewis number
$h_R°$ = heat of dissociation
α = atom concentration
H = total enthalpy
ϕ_c = catalytic efficiency
d = probe diameter

Subscripts

c = catalytic
nc = noncatalytic
e = edge of boundary layer
w = wall conditions
∞ = freestream conditions
2 = two-dimensional flow
3 = three-dimensional flow

Introduction

In hypersonic shock tunnels, high-pressure, high-enthalpy and highly dissociated gases are expanded in nozzles having large area ratios to get hypersonic Mach numbers. The expansion process gives rise to nonequilibrium effects, which make it difficult to obtain accurate measurements of flow quantities such as U, ρ, α, and T. Attempts have been made to predict the state of the flow along the nozzle centerline by measuring static pressure. However, since the static pressure is not as sensitive to nonequilibrium flow effects as atom concentration, an independent measurement of this quantity would throw more light on this phenomenon. A knowledge of the atom concentration is also essential for predicting the flow variables of the freestream in the nozzle.

The principle[1] is to arrange the experimental conditions so that the shock layer and boundary layer around the probe are frozen. If a probe containing catalytic and noncatalytic heat-transfer gages (herein to be called a differential heat-transfer gage) mounted side by side is placed in a dissociated hypersonic stream, then the freestream atom concentration can be determined from the measured values of heat transfer to the differential heat-transfer gage and the stagnation-point heat-transfer relation[2]:

$$(q_{c-nc}) = 2^{j/2} \times 0.54 S_c^{-0.63}(H_e - H_w) \times (\beta\mu_e\rho_e)^{1/2}(h_R°\alpha_\infty/H_e)\phi_c \quad (1)$$

where

$$\phi_c = 1/[1 + (S/k_w)]$$
$$S = 0.54 \times 2^{j/2}(\beta\mu_e\rho_e)^{1/2}S_c^{-0.63}\rho_w^{-1}$$
$$\beta = (2U_\infty/d)\{(\rho_\infty/\rho_e)[2 - (\rho_\infty/\rho_e)]\}^{1/2}$$

However, from Eq. (1) it is apparent that flow quantities like ρ_e, μ_e, and α_∞, etc. have to be known to obtain α_∞. Also, the catalytic efficiency ϕ_c of the silver-coated gage has to be determined independently. At present, there have been a few attempts[3,4] to measure the gage catalytic efficiency at very low speeds (50 to 100 fps). However, these experiments do not simulate the actual conditions that the catalytic gage is subjected to when placed in a hypersonic stream. The catalytic efficiency (ϕ_c) of the gage has to be measured in the actual environment in which the gage operates when it is used to obtain the freestream atom concentration. In addition, there is an uncertainty in the value of viscosity[5] of a dissociated gas at high temperature.

In order to avoid the previously mentioned difficulties in measuring atom concentration, the following method may be adopted so that the probe is self-calibrating with respect to its catalytic efficiency. Furthermore, by using this method, there is no need to know the viscosity, density, and flow velocity.

1. Theoretical Considerations

Equation (1) gives the general expression for differential heat transfer. Therefore, if two differential heat-transfer gage models (one three-dimensional and the other two-dimensional) are mounted side by side in a hypersonic stream and the differential heat transfer to both models is measured simultaneously, then the expressions for heat transfer are given by 1) three-dimensional flow

$$(q_{c-nc})_3 = 0.763 S_c^{-0.63}(H_e - H_w)(\beta_3\mu_e\rho_e)^{1/2}(h_R°\alpha_\infty/H_e)\phi_{c_3} \quad (2)$$

where

$$\phi_{c_3} = 1/[1 + (S_3/K_w)] \qquad S_3 = 0.763(\beta_3\mu_e\rho_e)^{1/2}S_c^{-0.63}\rho_w^{-1}$$
$$\beta_3 = (2U_\infty/d_3)\{(\rho_\infty/\rho_e)[2 - (\rho_\infty/\rho_e)]\}^{1/2}$$

and 2) two-dimensional flow

$$(q_{c-nc})_2 = 0.54 S_c^{-0.63}(H_e - H_w)(\beta_2\mu_e\rho_e)^{1/2}(h_R°\alpha_\infty/H_e)\phi_{c_2} \quad (3)$$

where

$$\phi_{c_2} = 1/[1 + (S_2/K_w)] \qquad S_2 = 0.54(\beta_2\mu_e\rho_e)^{1/2}S_c^{-0.63}\rho_w^{-1}$$
$$\beta_2 = (2U_\infty/d_2)\{\rho_\infty/\rho_e[2 - (\rho_\infty/\rho_e)]\}^{1/2}$$

In the preceding equations the velocity of the wall chemical reaction of the catalytic surface (k_w) is assumed to be the same for both gages. Also, the flow variables U, ρ, and α are assumed uniform in the core of the flow in which the two models are mounted.

Dividing Eq. (2) by Eq. (3) yields

$$(q_{c-nc})_3/(q_{c-nc})_2 \equiv \delta_{32} = (2\beta_3/\beta_2)^{1/2}(\phi_{c_3}/\phi_{c_2}) \quad (4)$$

S_2, S_3, and ϕ_{c_2}, ϕ_{c_3} are interrelated and can be expressed as

$$S_3/S_2 = (2\beta_3/\beta_2)^{1/2} \equiv (2d_2/d_3)^{1/2} \quad (5)$$

$$\phi_{c_3}/\phi_{c_2} = \phi_{c_3} + (1 - \phi_{c_3})(d_3/2d_2)^{1/2} \quad (6)$$

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Using Eqs. (5) and (6), Eq. (4) reduces to

$$\phi_{c_3} = (\delta_{32} - 1)/[(2d_2/d_3)^{1/2} - 1] \qquad (7)$$

From Eq. (7), ϕ_{c_3} can be estimated if δ_{32} is measured in a given experiment since d_2 and d_3 are known. However, for the particular value of $d_3/d_2 = 2$, ϕ_{c_3} cannot be found. Therefore, in Fig. 1 Eq. (7) is plotted over a wide range of the ratio d_3/d_2 to evaluate the best range of diameter ratios. Since δ_{32} is to be measured experimentally, it can be seen from Eq. (7) that it is better to have the value of δ_{32} as high as possible in order to obtain an accurate value of ϕ_{c_3}. It can be seen from Fig. 1 that, for large values of δ_{32}, diameter ratio d_3/d_2 should lie between 1.0 and 0.5. Ratios below 0.5 give rise to construction difficulties. The operating regime of the probe is shown by the hatched area in Fig. 1. Once ϕ_{c_3} is obtained in a given experiment, the velocity wall chemical reaction can also be estimated if the flow quantities U, ρ, μ, etc. are known accurately. However, in the present method an accurate determination of k_w is not necessary for measuring atom concentration.

2. Atom Concentration

A substitution of ϕ_{c_3} from Eq. (7) into Eq. (2) gives

$$(q_{c-nc})_3 = 0.763S_c^{-0.63}(H_e - H_w) \times (\beta_3\mu_e\rho_e)^{1/2}(h_R{}^\circ\alpha_\infty/H_e)\}(\delta_{32} - 1)/[(2d_2/d_3)^{1/2} - 1]\} \qquad (8)$$

α_∞ can be found from Eq. (8) if μ, ρ, and U_∞ are known accurately, since δ_{32} and $(q_{c-nc})_3$ are measured in a given experiment. As noted previously, there are uncertainties in estimating these quantities. To avoid this, the following approach may be used.

Consider the heat-transfer equation to the noncatalytic gage in the three-dimensional model; since $\phi_{nc} \approx 0$,

$$(q_{nc})_3 = 0.763P_r{}^{-0.63}(H_e - H_w)(\beta_3\mu_e\rho_e)^{1/2}[1 - (h_R{}^\circ\alpha_\infty/H_e)] \qquad (9)$$

Dividing Eq. (8) by Eq. (9) and rearranging

$$h_R{}^\circ\alpha_\infty/H_e = 1/[1 + \phi_{c_3}L_e{}^{0.63}(q_{nc}/q_{c-nc})_3] \qquad (10)$$

where ϕ_{c_3} is estimated using Eq. (7).

To find α_∞ from Eq. (10), all of the quantities are measured except Le and He. The Lewis number (Le) is usually taken as a constant for a given temperature,[6] and He can be obtained fairly accurately from the reservoir conditions in a given hypersonic shock tunnel. Consequently, by using this approach, it is not essential to measure the flow quantities or the velocity of wall chemical reaction (k_w) of the gage in order to obtain the freestream atom concentration.

Although the preceding analysis is valid strictly for blunt body flows with boundary layers that are separated from the shock wave by an inviscid shock layer, the analysis can also be extended to the cases when the flow Reynolds number is low enough to cause the now viscous shock layer and the boundary layer to merge. Using the Stanton number derived by Cheng,[7] a similar type of analysis has been done, and simple expressions for catalytic efficiency and freestream atom concentration have been derived. Thus this approach is applicable over a wide range of probe Reynolds number. This unique approach is being verified in the University of Toronto, Institute for Aerospace Studies 11- × 15-in. hypersonic shock tunnel.

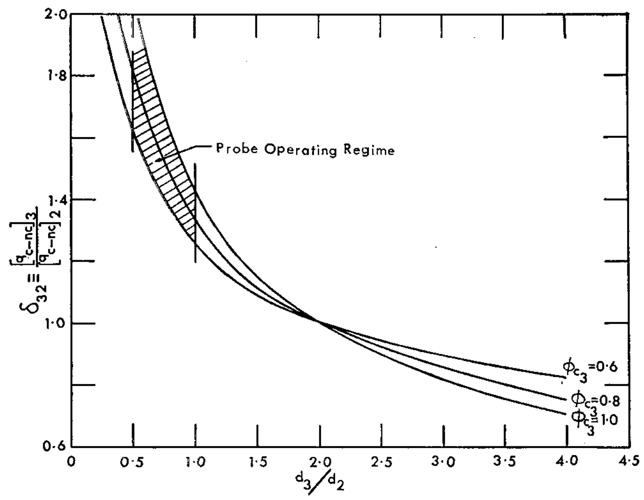

Fig. 1 Variation of probe sensitivity with diameter ratios.

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Nonzero Average Rate-Gyro Output from Sinusoidal Inputs

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Nomenclature

B	=	rate-damping coefficient
H_{IA}, H_{OA}, H_{SA}	=	angular momentum about the input, output, and spin axes
H_W	=	angular momentum of gyro wheel
I_{FIA}, I_{FOA}, I_{FSA}	=	moment of inertia of gyro float about input, output, and spin axes
K	=	electrical or mechanical spring constant
t_i	=	integration time
α_e	=	angular position error
Ω_{IA}, Ω_{OA}, Ω_{SA}	=	amplitude of sinusoidal angular rates about the input, output, and spin axes
θ	=	angular displacement between gyro float and case
ω_{EIR}	=	equivalent input angular rate
ω_{IA}, ω_{OA}, ω_{SA}	=	angular rates with respect to inertial space about the input, output, and spin axes
$\omega_n \equiv (K/I_{FOA})^{1/2}$	=	gyro natural frequency
ω_V	=	frequency of sinusoidal angular rates
$\xi \equiv \frac{1}{2}B(I_{FOA}K)^{-1/2}$	=	damping ratio

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